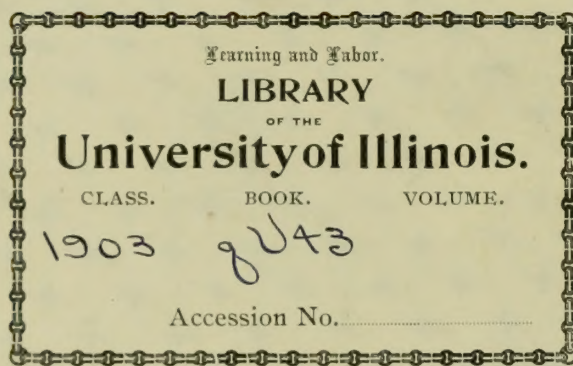


VARNES

Lateral Pressure
of Grain in Bins

Civil Engineering
B. S.

1903



Earning and Labor.

LIBRARY

OF THE

University of Illinois.

CLASS.

BOOK.

VOLUME.

1903

9043

Accession No.



1260
22
26 of 9

LATERAL PRESSURE

OF

GRAIN IN BINS

BY

ALBERT GRAFTON VARNES

THESIS

FOR

DEGREE OF BACHELOR OF SCIENCE

IN

CIVIL ENGINEERING

COLLEGE OF ENGINEERING

UNIVERSITY OF ILLINOIS

PRESENTED JUNE 1903



Digitized by the Internet Archive
in 2013

<http://archive.org/details/lateralpressureo00varn>

UNIVERSITY OF ILLINOIS

June 1, 1903

This is to certify that the thesis prepared under the supervision of Assistant Professor Ketchum by ALBERT GRAFTON VARNES, entitled LATERAL PRESSURE OF GRAIN IN BINS, is approved by me as fulfilling this part of the requirements for the Degree of Bachelor of Science in Civil Engineering.

Ira O Baker

Head of Department of Civil Engineering

61585

UNIVERSITY OF CALIFORNIA

June 1, 1906

This is to certify that the following named person

has been admitted to the University of California

and is entitled to the same privileges as other students

of this University.

Given at the University of California, the 1st day of June, 1906.

Attest: _____

President of the University of California

Secretary of the University of California

1100

Introduction: The object of this thesis is to investigate the problem of pressure of grain when stored in bins as in a modern grain elevator. Grain elevators have formerly been built mostly of wood, and since wood was comparatively inexpensive they have been built by rule of thumb, in most cases a large excess of material being used. High insurance rates on wooden elevators and grain stored in these elevators, together with the growing tendency to use, and the low cost of steel construction, have had a tendency to change the practice and many elevators are now being built of steel, and concrete and steel construction.

In order that the modern elevators may be designed economically and safely it is necessary that the laws governing the pressures and thrusts of stored grain be known.

The laws and formulas for the solutions of hydraulic problems are

not applicable to the case of grain which act somewhat as semi-fluids. Grain flows somewhat like a true liquid but differs from it in that there is friction between the different granules. The laws governing the pressure of stored grain are certainly similar to the laws of pressure of sand and earth as employed in the design of retaining walls and the investigation of the problem involves most of the difficulties met with in the latter case.

In the investigation of this subject, theories which are not based on carefully conducted experiments are of little value, and while on the other hand experiments made on a small scale may be very misleading.

Very few experiments have been made on the pressure of grain, and these are somewhat conflicting. The experiments by Roberts in England in about 1882 and by Prandtl in Germany in 1887-88 are the most reliable and will be discussed later.

Experiments on the flow of wheat through orifices by Willis Whited, (Proceedings of Engineering Society of Western Pennsylvania, April 1901) are interesting in this connection in that they show that the quantity of flow through an orifice is independent of the head of grain above the orifice.

The subject of pressure of stored grain will be discussed under the following heads: - (1), Experiments by previous experimenters; (2), Experiments by the author; (3), Discussion of experiments; (4), Formulas for the determination of the pressure of grain in bins; (5), Discussion of formulas; (6), Conclusion.

(1.) Experiments by Previous Experimenters

I Experiments on Pressure of Wheat in Bins by Isaac Roberts. (Engineering News, April 7, 1883.)

"Four cells were made of wood and numbered respectively 1, 2, 3, and 4.

4

Cell No. 1 was a hexagon, each side of which measured 4 inches, the inscribed circle being 7 inches in diameter. The height was 60 inches. The cell was supported on three legs, and the bottom consisted of a thin board about 9 inches square, resting upon a weighing machine, and counterpoised so as just to touch the bottom of the cell (when placed in position): half an ounce weight on the board would be indicated by the machine. The wheat was measured into the cell in imperial gallons and quarts, 1 gallon weighing 7.8 pounds. The weighing machine was weighted with 56 pounds, and after the given quantity of wheat had been measured into the cell, the weights were removed, half a pound at a time, till the machine indicated the pressure upon the movable bottom of the cell, within the limit of half a pound weight. One gallon of wheat poured into the cell measured $6\frac{1}{4}$ inches in height and indicated a pressure of $5\frac{1}{2}$ pounds on the bottom: two

gallons, $12\frac{1}{2}$ inches in height, pressure $7\frac{1}{2}$ pounds.

From two gallons up to and including nine gallons, or from $12\frac{1}{2}$ inches high in the cell to 56 inches in height, no increase in the pressure beyond $7\frac{1}{2}$ pounds was indicated, though nine gallons of wheat weighing 70.2 pounds were put into it. Twenty-six separate weighings were made with various quantities of wheat from two gallons up to ten gallons, and in no instance was the minimum pressure below 6 pounds, nor the maximum above $7\frac{1}{2}$ pounds. After obtaining the results above stated, I caused the cells Nos. 2, 3 and 4 to be made in order to determine if these results would be confirmed by square cells, and by larger hexagonal squares. Cell No. 2 was square, measuring 7 inches by 7 inches and 36 inches in height. The area was therefore equal to 49 square inches while the area of cell No. 1 was 41.59 square inches. Cell No. 2 was placed over the weighing

machine with a movable wood bottom, counterpoised in the manner already described.

Two gallons of wheat, measuring 11 inches in height, gave $8\frac{1}{2}$ pounds pressure on the bottom of the cell, and when it was filled with six gallons of wheat, weighing 46.8 pounds and measuring 33 inches in height, the pressure was still the same. With seventeen separate weighings of various quantities of wheat from two gallons up to seven gallons, the minimum pressure in any case did not fall below $7\frac{1}{2}$ pounds, nor the maximum rise above 9 pounds. Three gallons would sometimes give the maximum pressure, and six gallons would sometimes only weigh the minimum. The square cell No. 2, therefore, within narrow limits, confirms the results obtained by the hexagonal cell No. 1. Cell No. 3 was a hexagon measuring $6\frac{1}{2}$ inches on each side, the inscribed circle being 12 inches in diameter, and the cell 60 inches in height; the area

122.8 square inches.

This cell also was arranged over the weighing machine in the manner described for cells Nos. 1 and 2. When 11 gallons of wheat, weighing 87.97 pounds and measuring 24 inches in height were placed in the cell, the maximum pressure upon the bottom was reached, and any additional quantity of wheat up to 26 gallons weighing 202.8 pounds and filling the cell, did not increase the pressure upon the bottom.

Eighteen separate weighings were made with this cell, with varying quantities of wheat between 11 gallons and 26 gallons, and in any case the minimum pressure on the bottom was 46 pounds and the maximum 49½ pounds. The height of the wheat in the cell at 24 inches indicated these pressures. These results again confirm the accuracy of the data obtained by cells Nos. 1 and 2. Cell No. 4 was a hexagon, the sides of which measured 12 inches each. The inscribed circle 20¾ inches in diameter, height 96 inches.

The cell was placed over the weighing machine in the manner described with cells Nos. 1, 2 and 3; 51 gallons of wheat weighing 397.8 pounds, and measuring 36 inches in height in the cell, gave the maximum pressure of 238 pounds upon the bottom, and from 51 gallons up to 131 gallons, from 36 inches in height to 93 1/2 inches, or from 397.8 pounds up to 1,014 lbs. of wheat in the cell, the minimum pressure of 211 pounds, and the maximum of 238 pounds were obtained by the respective weighings. Nineteen trials with varying quantities of wheat were made. Again, the results obtained by the smaller cells Nos. 1, 2 and 3 are confirmed by this large one."

From these experiments Roberts proceeds to the following discussion: "It will be observed that all pressure upon the bottom ceases before the cells respectively are half filled with wheat. That the highest point within the cell where the pressure ceases, does not form the apex of a cone, having the diameter of the inscribed

Table of Results of Weighings

Cell Number	Diameter of Inscribed Circle	Area of Cell	Height of Maximum Pressure	Height of Cell	Mean Pressure on Bottom	Weight of Wheat in Cell
	in.	sq. in.	in.	in.	lbs.	lbs.
4	$20\frac{3}{4}$	374.11	36	96	224	1,014
3	12	122.8	24	60	48	202.8
2	7	49.0	11	36	$8\frac{1}{2}$	46.8
1	7	41.57	$12\frac{1}{2}$	60	7	70.2

circle at its base, is evident, but the wheat must form either a parabola or hyperbola with that base, and the whole of the weight of the wheat that may be filled into the cell, excepting only the parabola or hyperbola, is supported by friction against the sides, and the friction of the individual grains against each other. They form a self-supporting dome out of a plug of wheat held in position by friction. This generalization is borne out by each of the cells, whether it be large or small. The next step will be to determine some ratio existing between

the diameter or area of the cell and a portion of the axis of the parabola, the product of the length of which by the diameter or area will equal the cubical quantity of wheat that will represent the pressure upon the bottom. I find that 1.03 times the diameter of the inscribed circle, will furnish a constant that will give very fair results in each of the cases we are now considering, and be applicable in general practice.

The formulas are as follows:

Let A = area of cell in square feet

D = diameter of inscribed circle in feet.

C = 1.03 the constant.

W = weight of wheat in pounds per cubic foot.

P = pressure in pounds upon the bottom of cell.

$\frac{P}{A}$ = pressure per square foot of surface.

$$P = A \times D \times C \times W.$$

$$\frac{P}{A} = D \times C \times W.$$

In order to test the accuracy of the formula with the actual pressures as

determined in the four cells already described, we have the following:

P in cell No. 4 = 225.8 lbs.

The actual pressures, contained by nineteen weighings were:

211 lbs. minimum

238 lbs. maximum

224.5 lbs. the mean

P in cell No 3 = 42.83 lbs.

The actual pressure obtained by eighteen weighings were:

41.5 lbs., the minimum

49.5 lbs., the maximum

45.5 lbs. the mean

P in cell No. 2 = 9.7 lbs.

The actual pressure obtained by seventeen weighings were:

7.5 lbs. the minimum

9.5 lbs. the maximum

P in cell No 1 = 8.44 lbs.

The actual pressure given by twenty weighings were:

6 lbs. the minimum

The differences between the formulas and the actual weighings are within the limits of divergence that must always remain with wheat, for it is itself variable in specific gravity, in size of grains, in adhesiveness among the grains and in the quantity of moisture that is contained."

II Experiments on the Pressure of Grain by Prande, Engineer Nagel & Klamp, Hamburg.

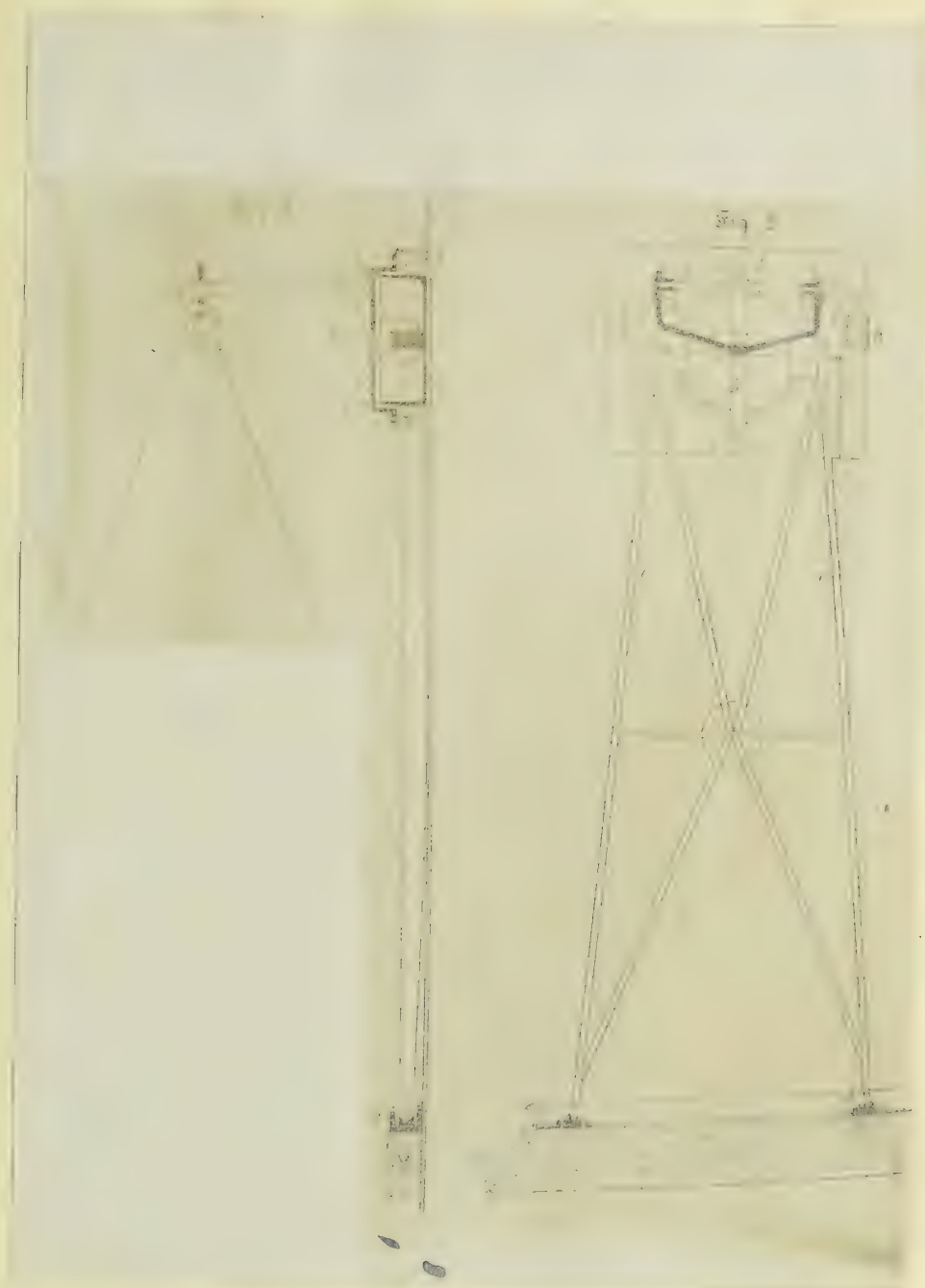
These experiments were made in 1887-88 by Prande, engineer for Nagel & Klamp, Hamburg. Two circular steel bins, 1.5 and 3.8 meters respectively in diameter were used in making the experiments. The lateral pressure for different heights was determined for the grain at rest and in motion. The following abstract is taken from an article by Prande in Zeitschrift Des Vereins Deutscher Ingenieure, Vol 40, 1896, pages 1122 to 1125.

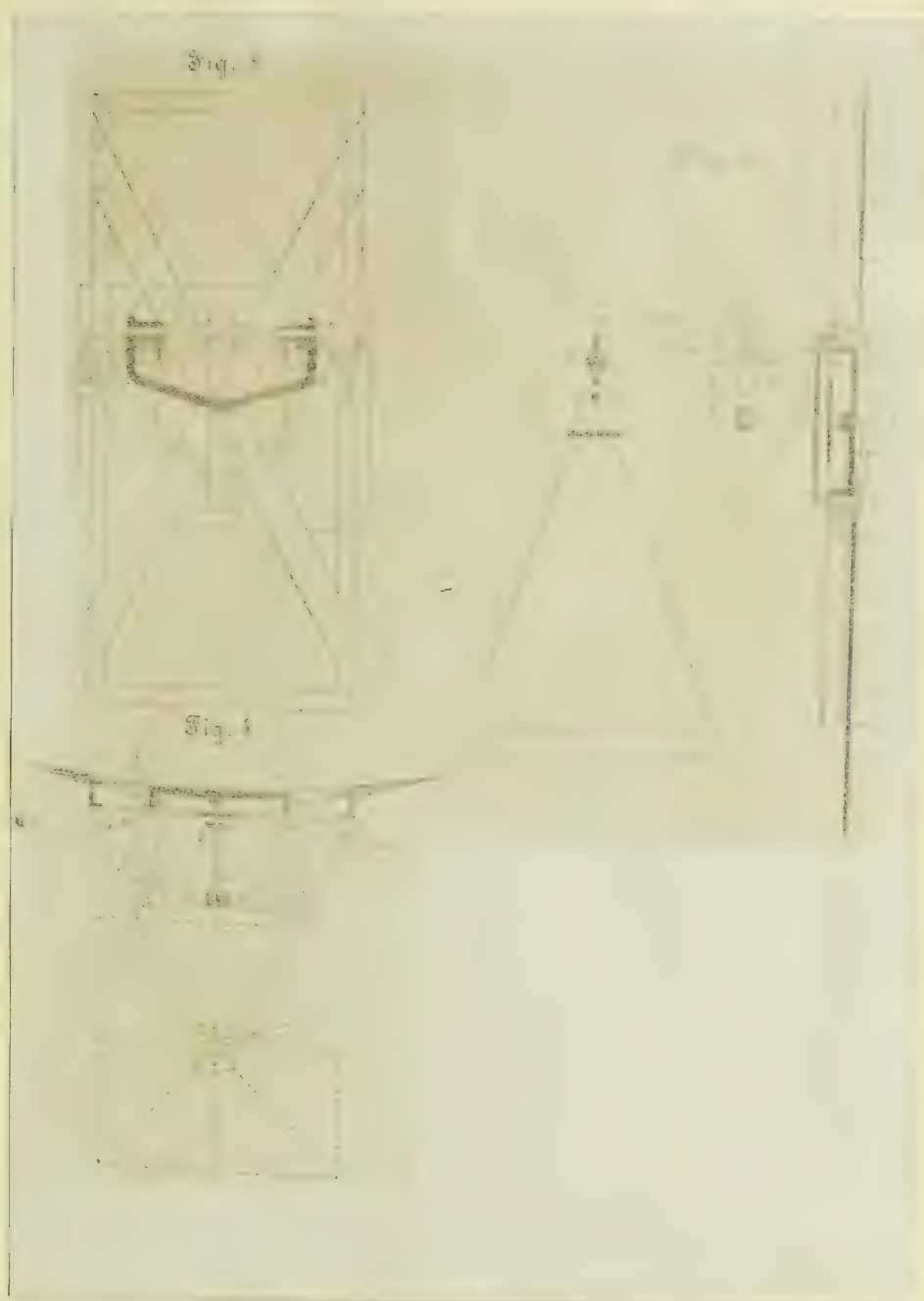
The Measuring Apparatus

"The apparatus first used is that shown in Figs. 1 & 2 which is built into an iron, cylindrically drilled plate b. It consists in the main of the cylindrical bell a, 200 mm diameter, which is carried by the frame c, which rests on the knife edge d. The bell can therefore move freely in the cylindrical opening of the plate b and without friction. The pressure of the grain against the bell is carried over on the scale beam e by a pin of hardened steel sharpened or pointed at one or both ends. This scale beam rests on two knife-edges of steel f, and carries a scale. The scale and beam are exactly balanced by the two weights h. Over the middle of the cylindrical opening is an iron hoop k with the two adjustable screws l, which determine the outward movement of the bell, while the inward movement is determined by two screws on nailed lists. In the tests, the screws l were so adjusted that the greatest movement of the bell was only 0.5 mm.

This apparatus proved very satisfactory taking into account the local difficulties, although it has some evident faults. First, the hanging of the scale beams t on the knife edges cannot be considered faultless. To prevent a dislocation of the knife edges by a movement of the bell, two frame members n (Fig 3, 4 & 5) were attached so that the pivot points were directly over the knife edges. A second fault is that the surface of pressure of the bell is not on the same level as its pivot of attachment and the resting point of the pin. On account of this a moment acts on the bell. It is very easy with cylindrical silos, where there is sufficient room to get the point of attachment of the bell and the pivot point of the pin at the same level with the pressure surface on the bell, but in Breunburg it was not possible.

Still it was possible to eliminate several faults in the apparatus first used, and therefore a second apparatus shown in Figs. 3, 4 and 5 was made.





LIBRARY
OF THE
UNIVERSITY of CHICAGO

The bell was made concave and the steel bars c so attached that the fastenings were on the same level as the point of application of the steel pins. The steel bars c were laid crosswise and riveted together acting as springs. But as it was impossible to put these points on the pressure level of the bell. There was still a moment which however was taken up by the steel bars c. To remove every doubt of the evil effect of the slightest overturning of the bell, even so small as not to be noticed by the eye, the cylinder was outwardly widened a little conical.

The Tests

Measurements were made on one silo 3.8 m diameter and one 1.5 m diameter. Great care was taken to keep the bell moving freely. The grain was put in with a bucket device provided with a measuring apparatus. When the grain was at the desired height it was leveled off and the vertical distance from the surface of the grain to the center of the pressure bell measured.

The weights were then removed one by one from the previously overloaded scale pan until the apparatus balanced. The pressure of the grain was then determined by the relation of $\frac{1}{2\frac{1}{2}}$, the ratio of the lever arms of the scale. To eliminate errors of observation pressures were taken with ~~a~~ gradually increasing heads and gradually decreasing heads.

To facilitate the observations in the crowded space, sheets of paper were put between the bell and the adjustable screws and when the proper number of weights were taken off these sheets were held in place by the pressure.

Tests were also made to ascertain if there is an increase of pressure against the walls when the grain is set in motion by letting grain run out at the bottom of the silo. It was observed that the slightest motion increased the pressure exceedingly. It was so great that the measuring apparatus had to be strengthened. Tables 1, 2 & 3 were taken with the first apparatus and

TABLE I.

Diameter of Bin = 12.26 ft

Height of Grain above Center of Bell Feet	Horizontal Pressure, pounds per sq. ft. Grain at Rest.			Remarks
	California Wheat	Russian Rye	Mixed Wheat 2 parts La Plata 1 part Russian 1 part Austrian	
3.28	9.6	12.3	13.1	
4.92	—	—	18.8	
6.56	16.3	24.5	—	
8.2	—	—	49	
9.8	22.4	40.7	65 to 74	
13.1	30.6	58.2	79 " 98	
16.4	43.8	81.5	102 " 122	
19.7	62.5	99.0	130 " 146	
23.0	77.5	118	138 " 163	
26.3	98.0	146	155 " 187	
29.6	110	175	163 " 171	
32.8	131	204	179 " 187	
36.1	164	233	196 " 204	
39.4	182	267	236 " 245	
42.7	204	298	262 " 302	
46.0	218	325	277 " 310	
49.2	243	357	326 " 392	
52.5	265	380	375 " 408	
55.8	—	—	408 " 438	

TABLE II.

Diameter of Bin = 12.26 ft.

<i>Head in Feet</i>	<i>Horizontal Press lbs. per sq. ft.</i>	<i>Velocity of Grain in Bin, ft. per sec.</i>	<i>Remarks</i>
3.28	12.2		
6.56	30.5		
9.85	48.2		
13.1	63.0		
16.4	75		
19.7	86		
23.0	98		
26.3	114		
29.6	124		
32.8	143		
36.1	153		
39.4	169		
42.7	185		
46.0	200		
49.2	218		
52.5	238		
55.8	257		
52.5	1162	.00108	
49.2	1141	"	
46.0	1040	"	
42.7	915	"	
39.4	815	"	
36.1	748	"	
32.8	658	"	

TABLE III

Diameter of Bin = 4.92 ft.

Pressure Head in Feet	Horizontal Pressure lbs. per sq. ft.		Remarks
3.3	27.7	to 32.7	
8.2	106	" 114	
13.1	171	" 180	Dry Wheat
18.1	196	" 213	2 parts La Plata
22.1	213	" 222	1 " Russian
25.7	228	" 237	1 " Austrian
29.2	242	" 253	
32.9	249	" 262	
36.5	262	" 269	
40.0	262	" 277	
44.7	269	" 287	
47.7	277	" 287	
50.8	277	" 287	
54.3	277	" 287	

TABLE IV

Diameter of Bin = 4.92 ft.

Pressure Head in Feet	Horizontal Pressure lbs. per sq. ft.	Velocity of Grain in Bin ft. per sec.	Remarks
19.7	225 to 327	.0013	
18.1	196 " 262	"	
18.1	90 " 114	—	
18.1	245 " 327	.0033	
14.1	237 " 327	"	
11.8	214 " 327	"	
9.8	214	"	
9.8	106 to 131	—	
9.8	237	.0033	
7.9	214 to 225	"	
5.9	143 " 163	"	
4.3	41 " 61	"	
2.5	31 " 32.1	"	
3.3	65 " 82	"	
5.2	90 " 106	"	
7.4	163	"	
9.2	196	"	
11.4	245	"	
13.1	316	"	
14.8	570	"	

Table 4 with the second.

For explanation of the tables ^{it} may be said that by the opening of the lower gate the contents of the silo were brought into motion, which, though small, was uniform, through each test. From the weight of the grain which ran out in a unit of time was calculated the velocity of discharge in the cylindrical part of the silo. These velocities did not exceed $1 \text{ m m } (\frac{1}{25})$ per second in the tests since for greater velocities the apparatus was judged to be too weak to stand the pressure. In each case the pressure head was kept constant while the weighing was being done, by discharging into the bin with the bucket apparatus just the amount that was being discharged at the bottom.

Prandl makes the following discussion of his experiments.

"The tests, however incomplete, have distinctly shown that the pressure of the grain against the wall of the silo is very small as long as the grain

is not in motion, but as soon as the grain is set in motion even with a small velocity it is sufficient to increase the pressure several times. It appears that with increasing velocity of the grain, the wall pressure approaches more and more that which a liquid would have of the same specific gravity as the grain."

III Experiments on the Flow of Grain through Orifices, by Willis Whited.

The following is an abstract of an article describing the experiments of Willis Whited, published in the Proceedings of the Engineering Society of Western Pennsylvania.

A number of experiments were made by Whited some years ago on the flow of grain through orifices under different heads. His results show and he concludes that the rate of flow is independent of the head. The grain seems to form a self supporting arch over the orifice and the "voussoirs" keep falling out through the opening below, being replaced by the grain from a column

directly above. This moving column runs vertically through the mass of grain and is supplied from the top. This explains the well known fact among grain men that the upper parts of the grain are discharged before the lower parts.

No matter how large the pressure head may be on the grain, it cannot be made to flow up through an opening. Even if the orifice is in the side, the grain will not flow out unless the inclination of the line passing through the top edge of the hole on the inside and the bottom edge on the outside is greater or at least nearly as great as the angle of repose. The flow through orifices of the same shape vary as the cubes of the diameters. Mr. Whited believes that the rate of flow is independent of the specific gravity of the granular mass. If the specific gravity is greater the friction increases so fast that the flow is not increased.

2. Experiments by Author

I Pressure of Wheat in Bins Experimental bin

The apparatus consists of a wooden box a b, Figs. 1, 2, 3 & 4, one foot square inside dimensions and eight feet high. This box is held in a suitable frame work c c' so that the bottom is one foot above the floor. The weighing apparatus d d' is arranged so that the pressure can be determined both on the bottom A and on the side B. The lever d, d, d' is used in measuring both bottom and side pressures. The arm d d' rests on a scale platform d'. d is a fulcrum and A & B are pressure surfaces. The ratio of the lever arms d d and d d' for bottom pressure was made 1 to 4 while the same ratio for side pressure was made 1 to 3. Some experiments were made in determining bottom pressure by setting the pressure surface A, directly on the scale floor.

Experiments on Pressure of Grain
In determining the pressure the

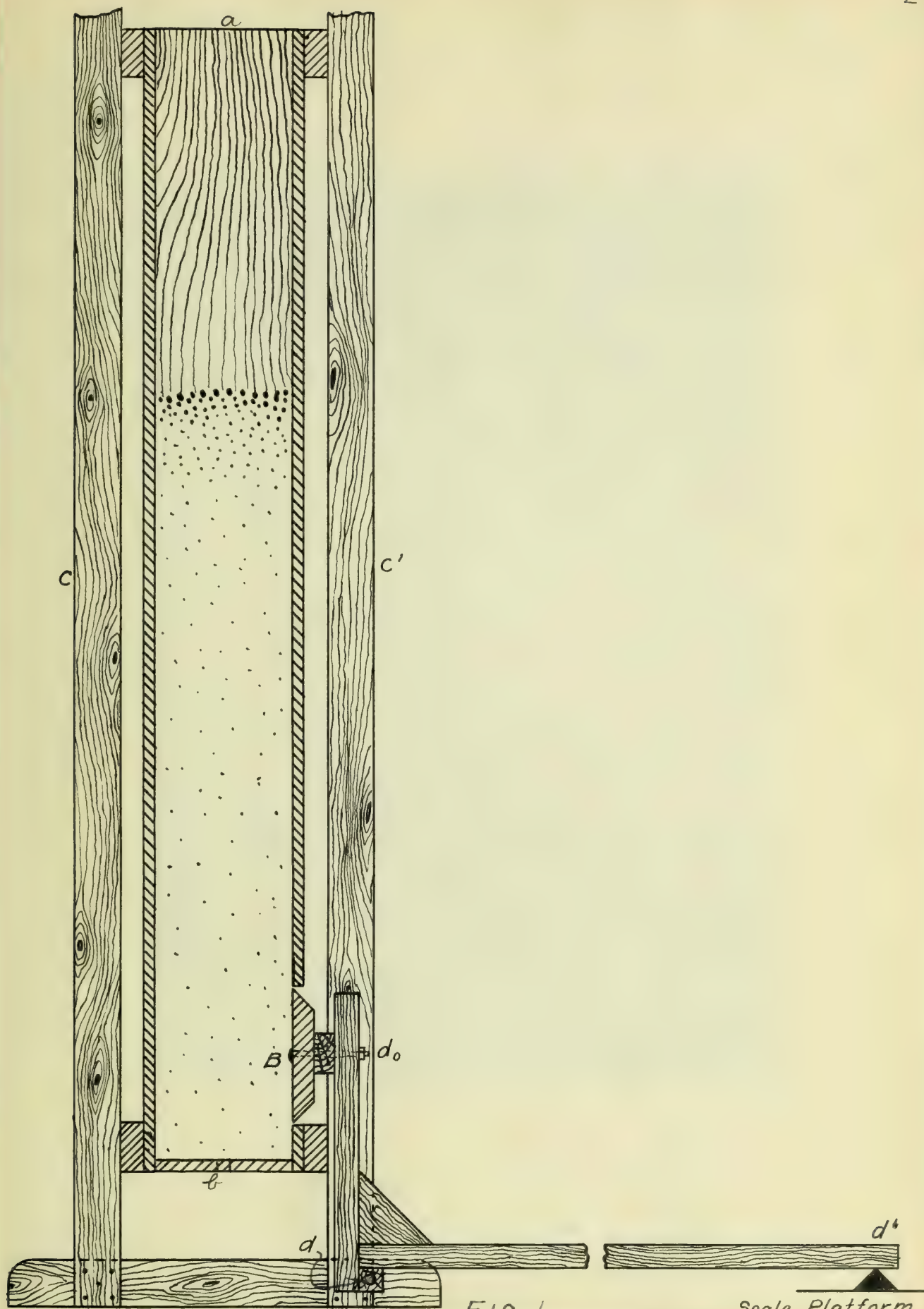


Fig. 1

LIBRARY
OF THE
UNIVERSITY of ILLINOIS



Fig. 2

LIBRARY
OF THE
UNIVERSITY of ILLINOIS.

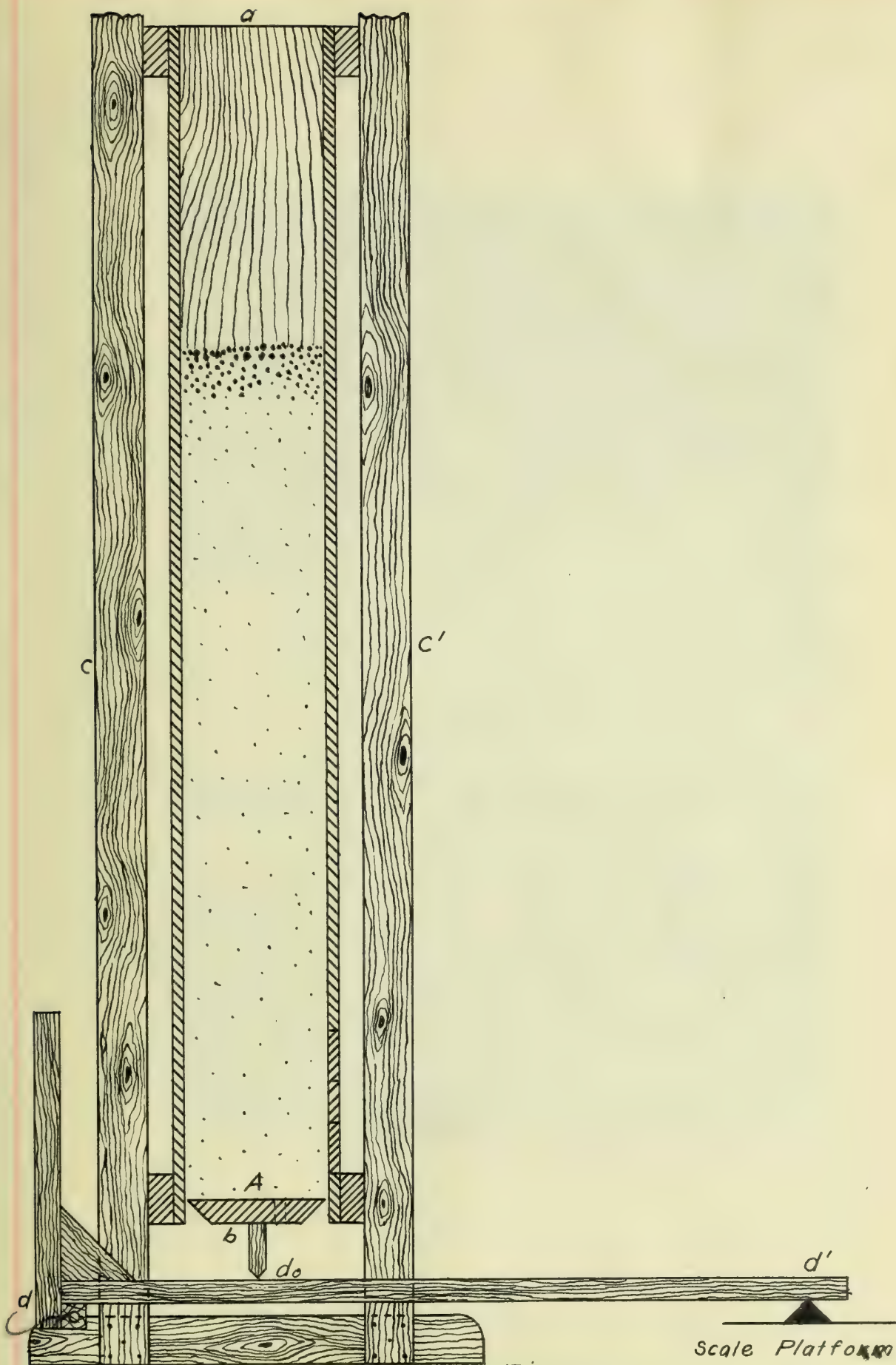


Fig. 3



Fig 4

LIBRARY
OF THE
UNIVERSITY of ILLINOIS.

bin was filled and the distance from the surface of the grain to the center of the pressure surface measured as pressure head. The grain ^{was} allowed to flow out at the bottom of the bin, between each weighing of the pressure for equal intervals of time, it having been determined by experiments that the flow of grain through an orifice is independent of the head. (See experiment on flow through orifices by the author.) The pressure head was again measured when the bin was nearly empty. The difference then, between the pressure heads divided by the number of intervals of flow gave the difference in head between each weighing and from this value the actual heads in each case was determined by simple addition. This method of determining the pressure heads is easier and more accurate than actual measurement.

No difference in pressure could be detected between when the grain in the bin was in motion, and at rest. In every case the pressure practically,

ceased to increase after the pressure head was equal to two or three times the width of the bin. The pressure frequently eased up suddenly as the grain was lowering in the bin. It would sometimes remain constant for three or four weightings and then all at once it would release 3 or 4 pounds. A peculiar thing occurred when the bottom pressure was measured directly on the scale floor. In some cases after enough weights had been put on the scale pan to start the beam down, it required as much as 30 or 40 pounds to pull the beam clear down. It was impossible to determine in that case what the real pressure was. It was therefore necessary to block the movement of the beam so that it could not ^{move} up and down over $\frac{1}{64}$ of an inch. When this was done the results agreed very well with those taken with the lever apparatus.

The data given in the tables were obtained in the order designated. Table I shows the pressure on the sides for

different heads. The area of the pressure surface is one square foot. It will be seen that although the pressure varies between the different heads, it does not begin to diminish appreciably until a head of as low as 1.9 feet is reached. From this point as the head diminishes the pressure falls off rapidly. The curve in Fig. 5 shows the relation of pressures to head determined in this experiment. In obtaining the results in Table II the experiment was begun with a head of 1.9 feet, and diminished 0.05 of a foot between each weighing. The curve in Fig. 5 shows the decrease of pressure. The two sudden releases in pressure were occasioned by moving the scales from the apparatus for the use of other parties. In these cases as in the case of pressure on the bottom, which will be noticed later, the pressure against the pressure surface was very much lessened whenever the pressure surface was allowed to move, as would be the result if the weighing apparatus

were disturbed. The variation in the results given in these two tables are not more than might be expected, considering the crudeness of the apparatus used.

Tables III, IV, V, VI and VII give the results of the experiments to determine pressure on the bottom of the bin. In all experiments except in that given in Table V the pressure surface was 1 foot square, filling the entire section of the bin. In the experiment giving the results shown in Table V, the pressure surface was a 9 inch circle. In the experiments giving the results shown in Tables III and IV, the lever apparatus was used while in the other experiments, the results of which are shown in Tables VI and VII, the pressure surface rested directly on the scale platform. As in experiments I & II to determine pressure on the side, the pressure on the bottom was practically constant above a head of two feet. Tables V & VI show the peculiar results of weighing the pressure directly on the scales, and not blocking down

the scale beam. It will be noticed that in some cases it required twice as much to pull the beam clear down as it did to start it, or vice versa, when the pressure surface was eased enough to allow the beam to come to the top the pressure on the pressure surface was diminished one half.

II Flow of Grain through an Orifice

Some experiments were made on the flow of grain through orifices. Circular orifices, one, two, two & one half and three inches in diameter were used, with heads up to 8.6 feet, in determining the flow through the bottom of the bin. The results are given in Table VIII. The amount of flow per second, fourth column of Table, was calculated for each experiment. Owing to the crudeness of the apparatus used, the time of beginning and end of flow could not be obtained closer than one half of a second. This error in observation explains the slight variation in the amount of flow per second under the different heads. The ratio of the amounts of

Table I
Pressure on Side

Head in Feet	Time of Flow	Pressure per sq.ft.	
		lbs.	oz.
8.0	15 sec	30	- 12
7.3		28	- 20
6.5		29	- 4
5.8		27	- 3
5.1	"	28	- 8
4.0		27	- 14
3.5	15 sec	30	- 0
2.7		28	- 8
1.9	17 sec	26	- 7
1.2	15 sec	24	- 12
0.8	15 sec	21	- 0
0.0		5	- 12
-1.0	"	0	

Area of Pressure surface = 1 sq. ft.

Pressure on Side Table II

Head in Feet on Center of Pressure Surface	Time of Flow	Pressure per sq.ft.	
		lbs.	oz.
1.9	15 sec	19	14
1.85	"	21	3
1.80	"	22	2
1.75	"	22	8
1.70	"	22	11
1.65	"	22	11
1.60	"	22	11
1.55	"	22	11
1.50	"	22	0
1.45	"	21	6
1.40	"	19	8
1.35	"	20	12
1.30	"	20	12
1.25	"	20	12
1.20	"	20	12
1.15	"	19	14
1.10	"	19	14
1.05	"	19	5
1.00	"	18	12
.95	"	18	6
.90	"	17	13
.85	"	17	4
.80	"	16	2
.75	"	12	6
.70	"	15	3
.65	"	13	11
.60	"	12	15
.55	"	12	6
.50	"	11	10
.45	"	10	11
.40	"	9	15
.35	"	9	3
.30	"	8	7

Area of Pressure surface = 1 sq. ft.

Head in Feet on Center of Pressure Surface	Time of Flow	Pressure per sq. ft. lbs. oz.
.25	15 sec	7 - 14
.20	"	6 - 6
.15	"	5 - 13
.10	"	4 - 11
.05	"	4 - 5
.00	"	3 - 6
-.05	"	2 - 7
-.10	"	2 - 7
-.15	"	1 - 14
-.20	"	1 - 14
-.25	"	1 - 5
-.30	"	1 - 5
-.35	"	0 - 8
-.40	"	0 - 7
-.45	"	0 - 3
-.50	"	0 - 3

Table III
Pressure on Bottom with Lever Apparatus

<i>Head in Feet</i>	<i>Time of Flow</i>	<i>Pressure in lbs. per sq. ft.</i>
8.6	30 sec.	80
7.98	"	80
7.36	"	77
6.74	"	77
6.12	"	77
5.50	"	76
4.88	"	75
4.26	"	74
3.64	"	73
3.02	"	72
2.4	"	72
2.28	1 min.	68
2.17	30 sec.	68
1.96	"	68
1.84	"	68
1.72	"	67
1.61	"	67
1.49	"	67
1.38	"	67
1.26	"	66
1.15	"	66
1.03	"	65
.92	"	65
.80	"	63
.69	"	62
.57	"	59
.46	"	54
.34	"	48
.23	"	42
.11	"	36
.0	"	29

Area of Pressure surface = 1 sq. ft.

Table IV
Pressure on Bottom with Lever Apparatus

<i>Head in Feet</i>	<i>Time of Flow</i>	<i>Pressure in lbs per sq. ft.</i>
8.6	30 sec.	72
7.94		74
7.28		74
6.63		74
5.97		74
5.32		74
4.66		74
4.01		74
3.35		73
2.7	1 min.	73
2.5		69
2.34		69
2.11		69
1.89		69
1.67		68
1.45		68
1.23		66
1.00		66
.78	30 sec	61
.55		51
.33		45
.22		38
.11		33
0		26

Table V
Pressure on Bottom weighed direct on Scales

Head in Feet	Time of Flow	Pressure in lbs. on 9" circle
8.6	^ 30 sec. " " " " " " " " " " " " Out at side	10 to 21
7.9		20 " 32
7.23		18 " 35
6.56		17 " 32
5.89		14 " 31
5.22		13 " 33
4.55		16 " 33
3.88		13 " 31
3.21		15 " 30
2.54		12 " 29
1.87		17 " 30
1.2		14 " 29
.96	X 1 min " " 30 sec " " " " " " Out at Bottom	13 " 21
.72		12 " 18
.48		11 " 17
.36		8 " 12
.24		7 " 11
.12		5 " 7
0		3 " 4

Note: Scale beam was not blocked
down. (See pp)

Table VI
Pressure on Bottom, direct on scales

Head in Feet	Time of Flow	Pressure in lbs per sq. ft.
8.6	^ 30 sec " " " " " " " " " " " " " " " " " "	74 to 91
7.9		56 " 90
7.23		54 " —
6.56		54 " 87
5.89		56 " 87
5.22		55 " 87
4.55		51 " 85
3.88		52 " 84
3.21		49 " 83
2.54		49 " 82
1.87	x 1 min " " " " " " " " " " " " " " " " " "	44 " 77
1.2		39 " 70
.96		34 " 54
.72		27 " 50
.48		24 " 40
.24		17 " 29
.12		6 " 18
.0		5 " 14

Table VII.
Pressure on Bottom, direct on scales

Head in Feet	Time of Flow	Pressure in lbs. per sq. ft.
8.6	^ 30 sec	102
7.90	"	95
7.23	"	89
6.56	"	86
5.89	"	83
5.22	"	81
4.55	"	79
3.88	"	77
3.21	"	71
2.54	"	69
1.87	"	65
1.2	"	62
.96	x 1 min	55
.72	"	52
.48	"	45
.24	"	30
.12	"	22
0	"	15

Table VIII
Flow through Orifices

Head at Beginning of Flow	Time of Flow	Amount of Flow lbs. oz.	Flow per sec. lbs.	Ratio of Flow compared with 1" Orifice	Cubes of Diameters
3 inch					
9	15 sec	64 - 14	4.32	24.2	27
7.7	"	65 - 12	4.38		
6.4	"	63 - 3	4.21		
5.1	15½ sec	68 - 13	4.44		
3.8	14½ sec	61 - 14	4.20		
2.5	15 sec	64 - 18	4.32		
1.2	14 sec	61 - 4	4.37		
2½ inch					
9	30 sec	73 - 14	2.46	14	15.6
7.36	"	75 - 5	2.51		
5.72	"	76 - 0	2.53		
4.08	"	75 - 6	2.51		
2.44	"	77 - 10	2.58		
0.80	15 sec	36 - 11	2.45		
2 inch					
9.0	1 min	81 - 4	1.35	7.5	8
7.32	"	80 - 13	1.34		
6.64	"	81 - 2	1.35		
3.96	"	81 - 3	1.35		
2.28	"	80 - 9	1.34		
0.60	20 sec	27 - 15	1.39		
1 inch					
9	7 min	75 - 10	0.1800	1	1
7.43	"	75 - 13	0.1805		
5.86	"	75 - 13	0.1805		
4.29	"	75 - 15	0.1807		
2.72	"	75 - 8	0.1800		
1.25	5 min	53 - 6	0.1780		

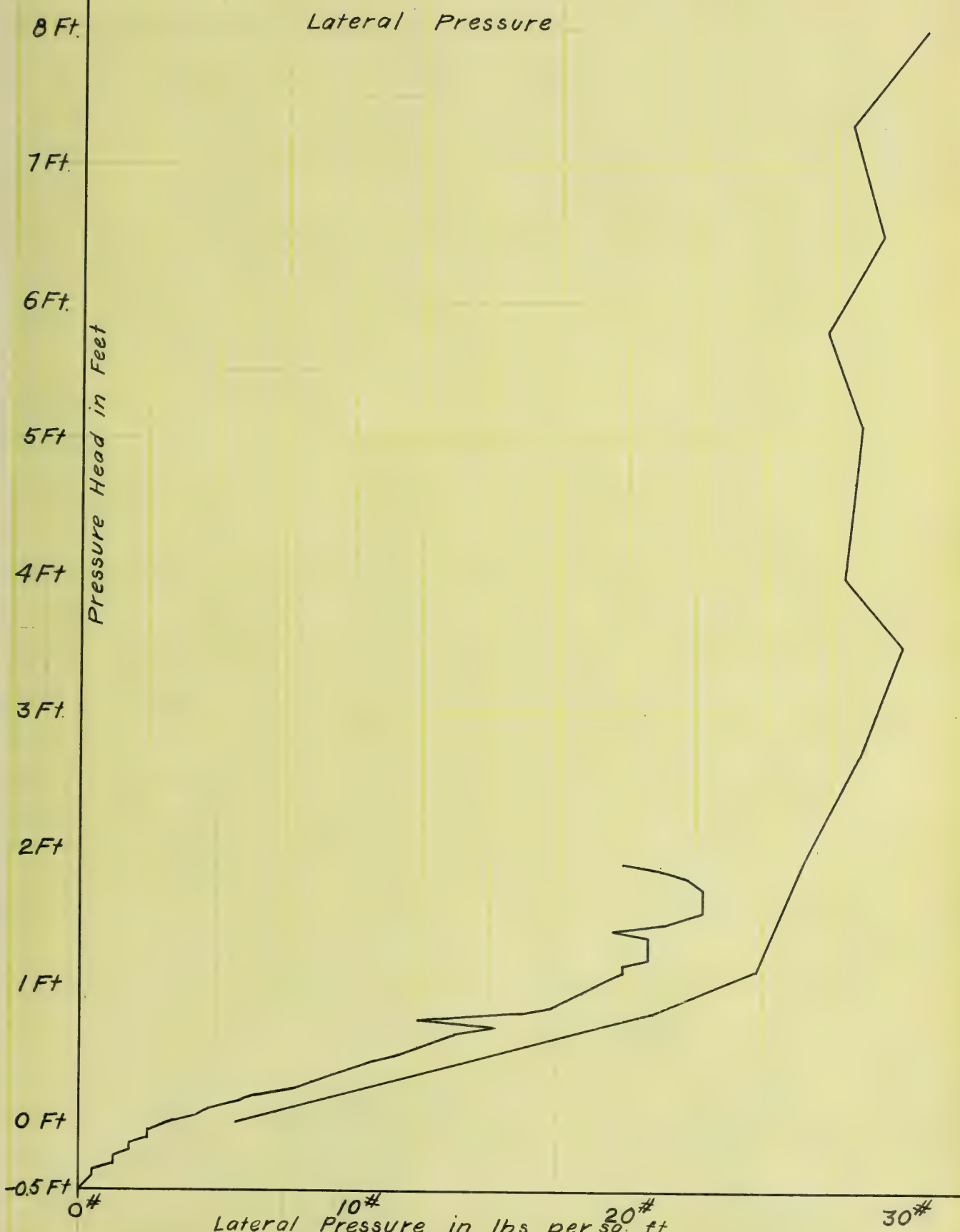


Fig. 5

Bottom Pressure

8 Ft.
7 Ft.
6 Ft.
5 Ft.
4 Ft.
3 Ft.
2 Ft.
1 Ft.
0 Ft.

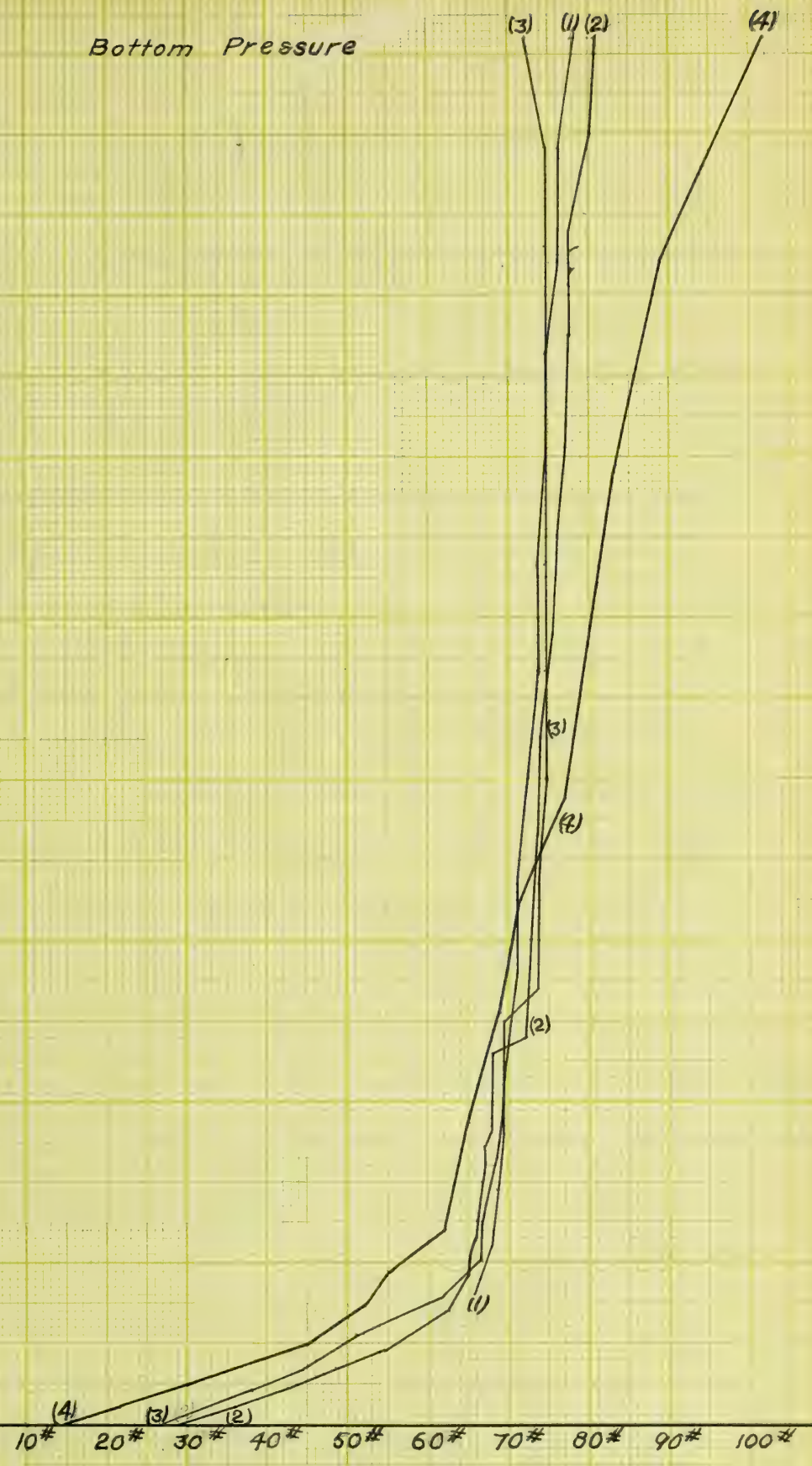


Fig 6

47

flow through the different orifices compared with the one inch orifice is also shown in Table VII

3 Discussion of Experiments

The preceding experiments all show that when the grain is at rest the pressure practically ceases to increase after the height of the grain has reached two or three diameters of the bin. Robert's experiments show no increase of pressure after a height of two or three diameters was reached, but the experiments of Prandl, which were on a much larger scale, and those of the author show a slight increase of pressure with an additional increase in head. All of these experiments show that in tall bins of small cross-section a very small per-cent of the weight of the grain comes on the bottom and that almost the entire weight of the grain is carried by the sides. This conclusion is supported by the behavior

of bins in actual use. Large bins constructed on the usual plan of building the walls of planks laid flatwise one on top of the other, have been shortened as much as one foot in height in eighty feet, by compression of the sides. A bin shortened in this way would surely bring undue strains on the bottom of the bin. This offers an explanation for the failure of grain bins by the breaking out of the bottoms. In all the experiments that have been made the walls of the bins were not compressible to any extent, and therefore the results obtained for bottom pressures are no doubt misleading when applied to elevator bins with compressible sides. If the sides are compressed the pressure on the bottom will approach that which would be given by a liquid. The self supporting arches in the upper parts of the bin, being relieved at the haunches by the settlement of the walls can no longer support the load above them, and thereby bring

41

an increased load on the arches in the lower part of the bin, increasing the pressure there, both on the bottom and on the sides. Viewing it in another way it is just trying to force the bottom up through the mass of the grain. In the design of bins particular care should be given to the design of side walls to prevent shortening.

Grains when stored in a bin seems to form self supporting arches which span the width of the bin and are supported at the haunches by the friction on the sides due to lateral pressure. On the other hand the lateral pressure is occasioned by the loading on these arches, which, being limited by the arches above, limits the lateral pressure. Now if the grain in the bin were set in motion by opening an orifice at the bottom, a great number of the arches would be broken bringing thereby a heavy load on the arches below and thus increase the lateral pressure.

many times that due to quiescent grain
Pranley's experiments on grain in motion
show this to be true. With a very slight
motion of the grain in his experiments
he obtained pressures five or six times
as great as when the grain was at rest.
His experiments were on a large scale
compared with any that have yet been
performed and therefore should be given
proportional weight. In the experiments
performed by the author no increase
in pressure was noticable with the grain
in motion, except when the flow was
suddenly stopped. In this case there
was a sudden upward movement of the
scale beam for an instant, the amount
of which could not be readily weighed.
This action of the moving grain was
similar to water hammer. The fact
that there was no increase in pressure
when the grain was in motion may
possibly be due to the small scale upon
which the apparatus was built.

The experiments of the author on
the flow of grain through orifices
agree very well with those performed

by White. In all the experiments on flow through orifices as well as those on pressure, an excellent grade of Illinois winter wheat was used. The grains were plump and the wheat weighed 60 pounds per bushel. White's statement that the flow through an orifice varies as the cube of the diameter and is independent of the head is fully supported by the author's experiments. (See Table VIII pp. 44). From the tables the following formula may be deduced for the flow of grain through circular orifices:

$$q = 0.18 t d^3$$

where q = discharge of wheat from orifice in pounds.

t = time in seconds

d = diameter in inches

Errors in calibration explain the variation of the results of the experiments from those given by the formula.

4. Formulas for the Determination of Grain Pressure in Bins.

Several formulas have been

deduced for the lateral pressure of grain. All of the formulas that have been deduced so far are for grain at rest. Roberts in his discussion gives the following formula for the pressure on the bottom:

$$P = A \times D \times C \times W$$

where A = area of section of cell in square feet.

D = diameter of inscribed circle in feet.

$C = 1.03$ the constant

W = weight of wheat in pounds per cubic foot.

Rankine's Formula, Eng. News Vol. 38 pp. 427.

$$P = W \times \frac{1 - \sin \phi}{1 + \sin \phi}$$

where P = pressure in lbs per sq ft.

W = weight per cubic foot

x = depth from surface

ϕ = angle of repose of grain.

Trantwine's Formula

Trantwine, pp 608, 10th edition.

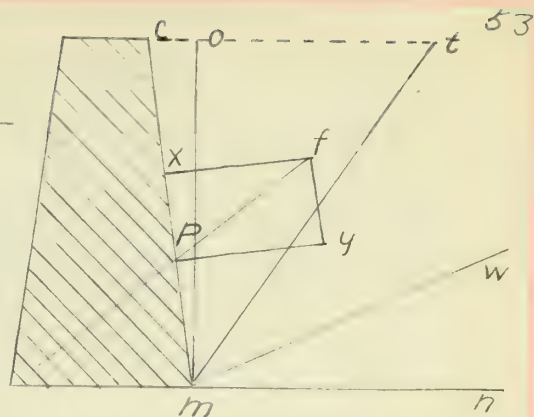
$$\text{Pressure } fP = \frac{\text{wt. of } \triangle cmt \cdot ot \cdot \sec \angle yPf}{\text{vertical depth } om}$$

$$= \frac{\text{wt. of } \triangle cmt \cdot ot}{\cos \angle yPf \cdot om}$$

where $\angle Wmr = \text{angle of repose}$
 mt bisects $\angle cmw$

$\angle yPf = \text{angle of wall friction.}$

This formula is used by the Link Belt Engineering Co. in the design of coal bins.



Turner's Formula

The following formula is used by Mr. C. A. P. Turner, M. Am. Soc. C. E., Consulting Engr., Minneapolis, Minn.

$$P = 16 h$$

where $P = \text{pressure in lbs per sq ft.}$

$h = \text{head in feet}$

Mr. Turner has used this formula in designing steel elevators of 371 millions of bushels capacity. He has a patent on a system of vertical angle bracing which is put on the inside of circular steel bins to strengthen the side walls. Although he uses no

formula. for computing compression in the sides, he does acknowledge that there is compression there in putting in the vertical strengthening angle.

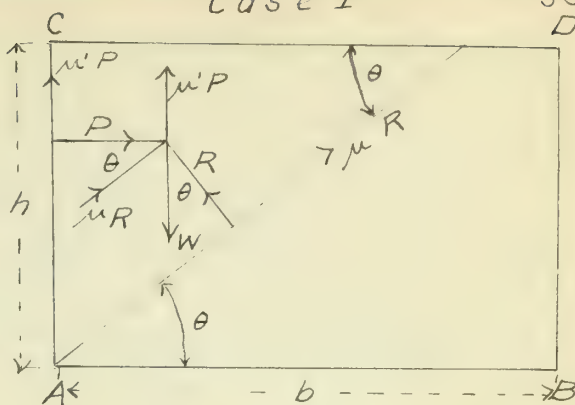
Airy's Formula, Proceedings of Institution of Civil Engineers, London. Vol. CXXXI, pp 347.

Airy considers two cases in determining lateral grain pressure, case I, when the depth of grain is such that the plane of separation which produces the maximum lateral pressure, intersects the surface of the grain inside the opposite wall, and case II, when this plane of separation intersects the surface of the grain outside of the opposite wall. The formulas are for pressure against a retaining wall based on Rankine's solutions. The proof is given on pp. 354, appendix to the article mentioned above.

Case I

$$P = \gamma \cdot \frac{h^2}{2 \tan \theta} \cdot \frac{\tan \theta - \mu}{2(1 - \mu \mu') \tan \theta + (\mu + \mu') \tan \theta} \quad (1)$$

55
D



h^u = height of bin

b - breadth of bin, y = wt. of cu. ft. grain

θ = angle that plane of separation makes with the horizontal.

μ = coefficient of friction of grain on walls of bin.

μ' = coefficient of friction of grain on grain.

That value of θ must be used that will give a maximum pressure, hence the first differential is equated to zero, and the value solved for $\tan \theta$ which gives :

$$\tan \theta = n + \sqrt{\frac{n^2 + n'^2}{n + n'}} \quad (2)$$

Using the value of $\tan \theta$ from (2) in (1) the pressures for case I may be calculated.

Case II.

$$P = \gamma \cdot \frac{b}{2} (2h - b \tan \theta) \frac{\tan \theta - \mu}{1 - \mu \mu' + (\mu + \mu') \tan \theta} \quad (3)$$

The value of $\tan \theta$ which gives

maximum P , found by equating first differential equal to zero is:

$$\tan \theta = \sqrt{\frac{2h}{b} \cdot \frac{1+\mu^2}{\mu+\mu'} + \frac{1+\mu^2}{\mu+\mu'} \cdot \frac{1-\mu\mu'}{\mu+\mu'} - \frac{1-\mu\mu'}{\mu+\mu'}} \quad (4)$$

The curve in Fig 7 pp 50 gives the total pressure on the side of a bin on one ft in length, and was obtained from the preceding formulas in the following manner:

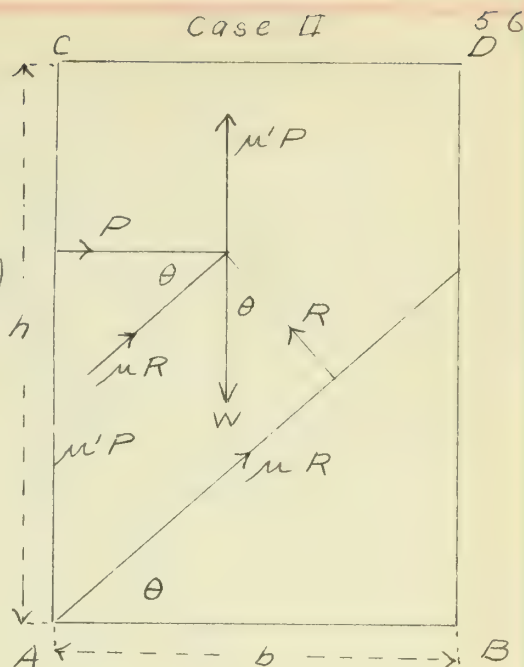
Equations (1) and (3) were divided by b^2 giving:

$$\frac{P}{b^2} = \gamma \cdot \frac{h^2}{2b^2 \tan \theta} \cdot \frac{\tan \theta - \mu}{1 - \mu\mu' + (\mu + \mu') \tan \theta} \quad (1a)$$

$$\frac{P}{b^2} = \gamma \cdot \left(\frac{h}{b} - \frac{\tan \theta}{2} \right) \frac{\tan \theta - \mu}{1 - \mu \mu' + (\mu + \mu') \tan \theta} \quad (2a)$$

The curve is plotted by solving (1a) up to the point when $\tan \theta = \frac{b}{a}$, and by solving formula (3a) from there on

The unknowns in these equations are $\frac{h}{b}$ and θ . By substituting different values of $\frac{h}{b}$ in (4), values of $\tan \theta$



are obtained and by substituting corresponding values of $\frac{h}{b}$, and $\tan \theta$ in (3a) values of $\frac{P}{b^2}$ are obtained. Different values of $\frac{P}{b^2}$ were calculated and plotted. Values of $\tan \theta$ from (2) were used in (1a) for values from zero up to $\frac{h}{b}$. To obtain the pressure on a unit surface for any given depth, find the total pressure on two depths varying by one foot. The difference between these two values will give the pressure on a unit surface.

Pressures were calculated from this curve, Fig. 7 pp. 53 for the bin used by the author in his experiments. A comparison of the calculated pressures with the pressures determined by experiment, is shown by the curves in Fig 8 pp. 54. The two agree almost exactly for the smaller heads and for the larger heads the formula is on the safe side. Fig. 9 is a comparison of this formula with Prandl's experiments. Here again the formula is always on the safe side, and agrees almost exactly for

Pressure of Wheat in Bins

P = pressure in lbs. per lin. ft. of bin
 h = height of bin in ft.
 b = width of bin in ft.

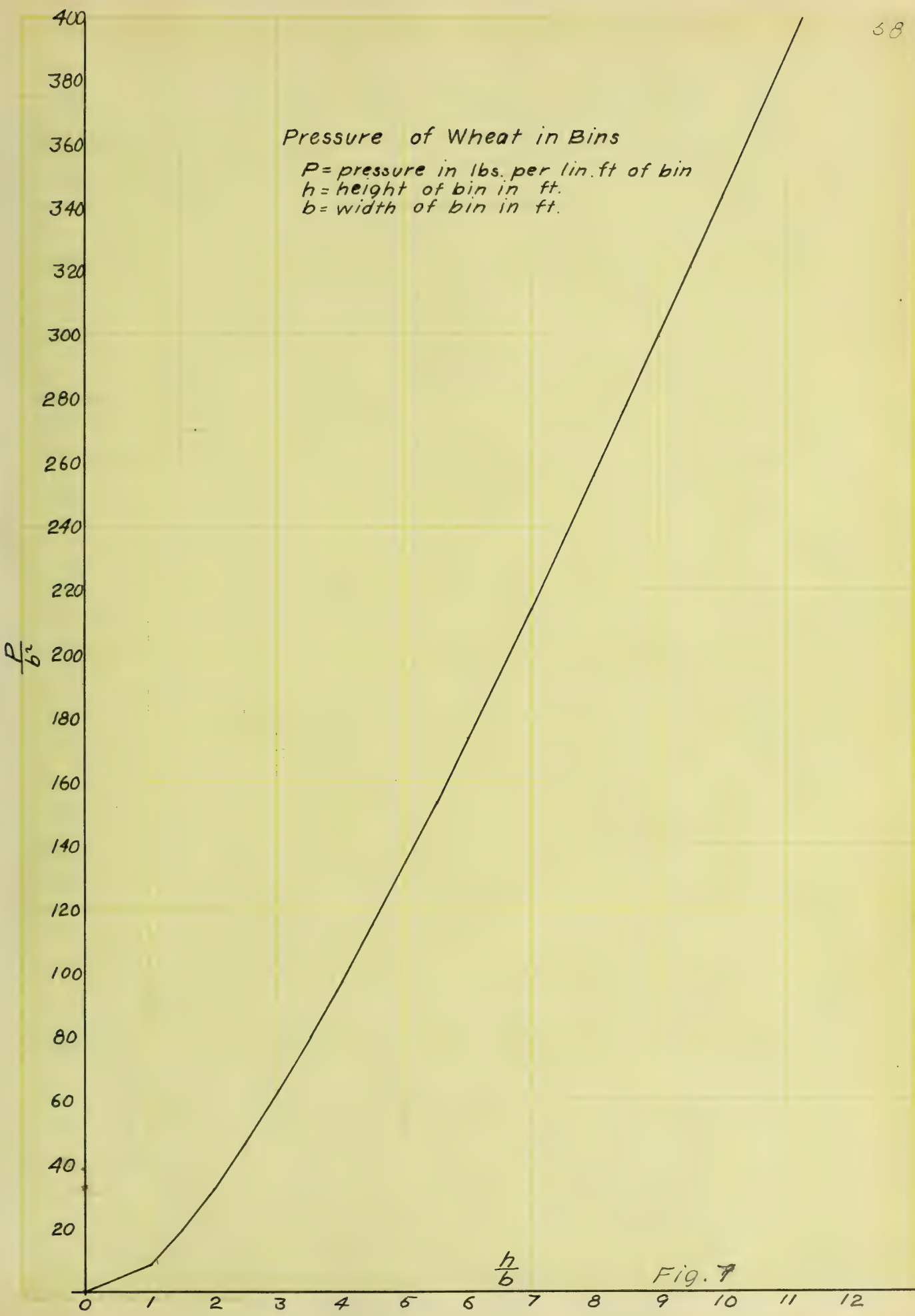


Fig. 7

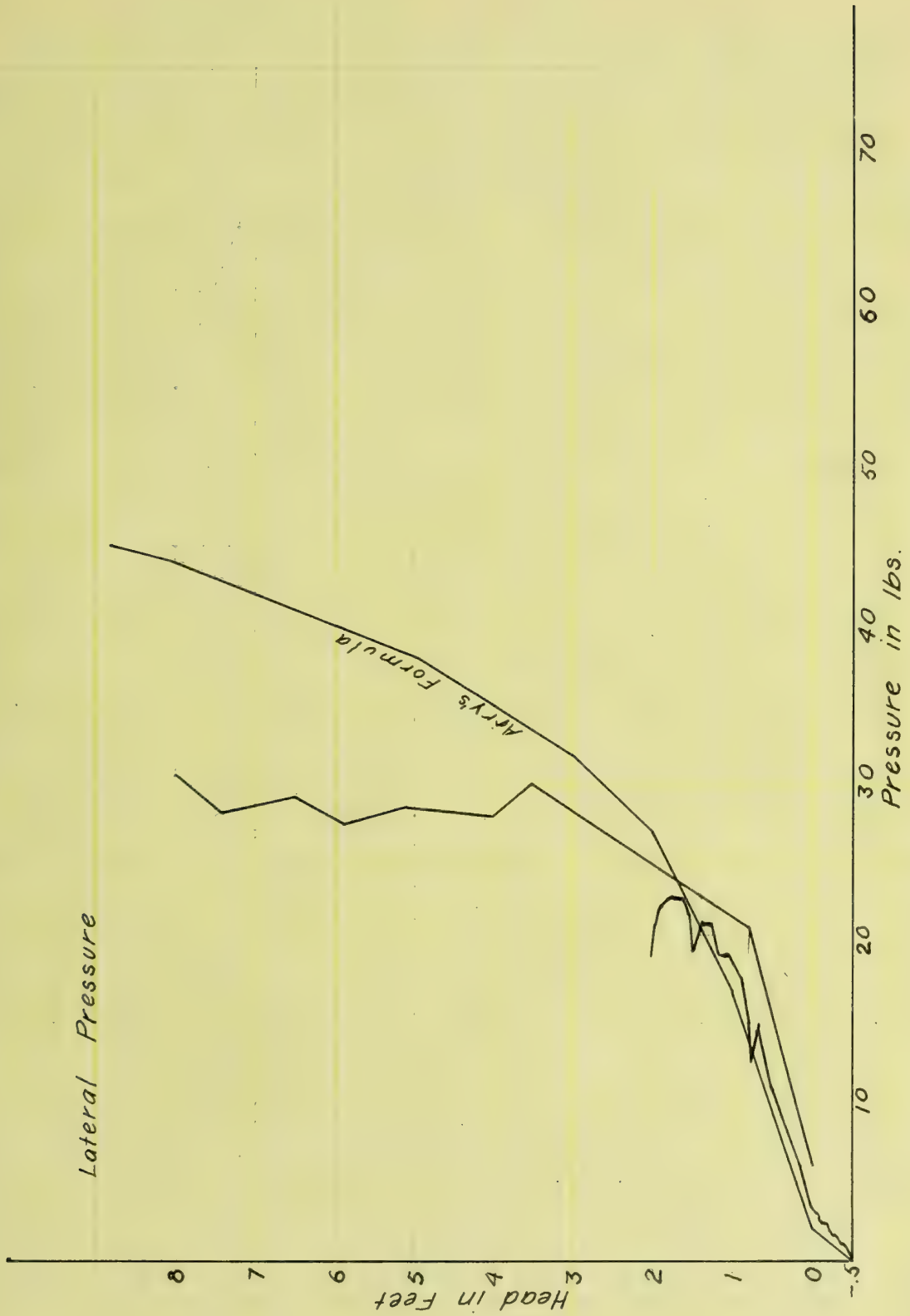
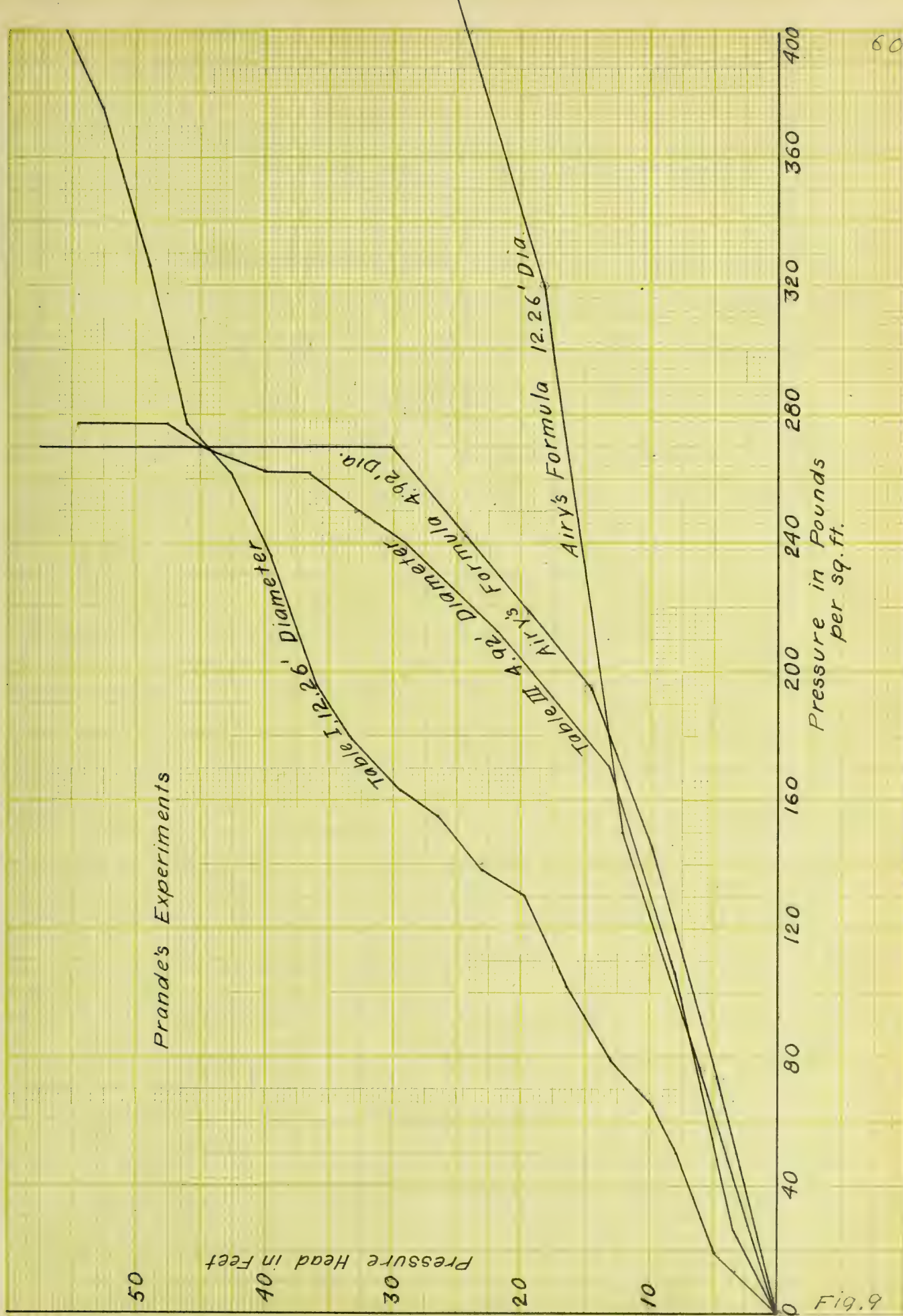


Fig. 8



the smaller bin.

61

5 Discussion of Formulas

Experiments and observations show that any rational formula for lateral grain pressure should take into account the following: (1) specific gravity of grain; (2), angle of repose or angle of friction of grain; (3), coefficient of friction of grain on walls of bin; and (4) the ratio of the height to the width of the bin.

Rankine's formula has no terms involving the ratio of height to width, nor the coefficient of friction of the grain on the walls of the bin. It does consider the specific gravity and the angle of repose of the grain.

Trantwine's formula has terms for angle of repose, wall friction, and specific gravity, but none for the ratio of height to breadth.

The formula used by Turner, which is only a modification of the hydraulic formula, considers

only specific gravity of the grain and the height. Turner probably takes account of vertical thrust independently of this formula.

Airy's formula is ideal in that it has all the elements which have been observed to affect lateral pressure in the experiments that have been performed. Consider for example formula (3) pp. 55 which gives the lateral pressure when the plane of separation that gives the maximum pressure intersects the surface of the grain outside the walls of the bin, as is the case generally in the bulk bins now in use. It contains the ratio of height to breadth, the angle of repose of the grain, the friction of the grain on the wall of the bin, and the weight of the grain.

6. Conclusion

From the experiments that have so far been performed the author

has arrived at the following conclusions:

(a.) Flow of grain through an orifice is independent of the head and varies as the cube of the diameter.

(b.) Pressure on bottom increases up to a height of 2 or 3 diameters of the bin and then remains constant.

(c.) Prandl's experiments show that the pressure of moving grain is several times as much as that of grain at rest. This fact however was not proved by the author's experiments. Experiments on a larger scale are necessary to determine this.

(d.) There is a compression in the sides of the bin due to the lateral pressure and friction. This is equal to the entire weight of the grain above a height equal to two or three diameters of the bin.

(e.) Of all the formulas yet proposed, Airy's seems the best. It agrees very well with the experiments that have been made.

If there is no increase in pressure on the sides or bottom when

the grain in a bin is in motion, this problem of grain pressure is solved. Ary's formula agrees with the experiments that have been performed almost exactly. Where there is any difference the formula is on the safe side. The formula may be easily applied to any case with the assistance of the curve. (Fig. 7, pp. 58)

The problem now is to decide whether or not there is an increase of pressure when the grain is in motion. To decide this question the experiments must be carried out on a large scale. To future experimenters the author advises that Prandl's policy of constructing a weighing apparatus to determine pressure in some large elevator be followed. The only expense in this case is the cost of putting the weighing apparatus in place. It is not necessary that the pressure surface be more than one foot square as the author's experiments on bottom pressure (Tables V & VI, pp. 41 & 42) show that the

pressure on any surface varies directly as the area. When this problem of pressure, when the grain is in motion, is solved the design of grain bins will be put on a rational basis

UNIVERSITY OF ILLINOIS-URBANA



3 0112 086762595